Chapter 2

The Time Value of Money

Time Discounting

One of the basic concepts of business economics and managerial decision making is that the value of an amount of money to be received in the future depends on the time of receipt or disbursement of the cash. A dollar received today is more valuable than a dollar to be received in the future. The only requirement for this concept to be valid is that there be a positive rate of interest at which funds can be invested.

The time value of money affects a wide range of business decisions, and a knowledge of how to incorporate time value considerations systematically into a decision is essential to an understanding of finance. This chapter is devoted to describing the mathematical models of compound interest. The objective is to develop skills in finding the present equivalent of a future amount and the future equivalent of a present amount.

The Interest Rate

A dollar available today is more valuable than a dollar available one period from now if desirable investment opportunities exist. There are two primary reasons why real investments can generate an interest return:

1. Some types of capital increase in value through time because of changes in physical characteristics, for example, cattle, wine, and trees.
2. There are many work processes where roundabout methods of production are desirable, leading to increased productivity. If you are going to cut down a large tree, it may be worth investing some time to sharpen your axe. A sharp axe may result in less time being spent cutting down trees (including sharpening time) than working with a dull axe. If you are going to dig a hole, you might want to

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build or buy a shovel, or even spend the time to manufacture a backhoe if it is a big hole. The investment increases productivity sufficiently compared to the alternative methods of production without capital so that the new asset can earn a return for the investor.

These characteristics of capital lead to a situation in which business entities can pay interest for the use of money. If you invest \$1 in an industrial firm, the firm may be able to pay you \$1 plus interest if your investment enabled the firm to use some roundabout method of production or to delay the sale of an item while it increased in value.

Future Value

Assume that you have \$1.00 now and can invest it to earn r interest. After one period, you will have the \$1.00 plus the interest earned on the \$1. Let FV be the future value and r be the annual interest rate. Then,

$$FV = 1 + r.$$

Repeating the process, at time 2 you will have

$$FV = (1 + r) + r(1 + r) = (1 + r)^2$$

and the future value of \$1.00 invested for n periods is

$$FV = (1 + r)^n.$$

If $r = 0.10$ and $n = 2$, we have

$$FV = (1 + r)^n = (\$1.10)^2 = \$1.21.$$

If instead of starting with \$1 we start with a present value, PV, of \$50, the value at time 2 is

$$FV = PV(1 + r)^n \tag{2.1}$$
$$FV = 50(1.10)^2 = \$60.50.$$

With a 0.10 interest rate, the \$50 grows to \$55 at time 1. The \$55 grows to \$60.50 at time 2. Equation (2.1) is the standard compound interest formula. The term $(1+r)^n$ is called the accumulation factor.

The power of compounding (earning interest on interest) is dramatic. It can be illustrated by computing how long it takes to double the value of an investment. Table 2.1 shows these periods for different values of r.

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Table 2.1. The Power of Compounding.

Interest Rate (r)	Time Until Initial Value Is Doubled
0.02	35.0 years
0.05	14.2
0.10	7.3
0.15	5.0
0.20	3.8

A useful rule of thumb in finance is the "double-to-72" rule, where for wide ranges of interest rates, r, the approximate doubling time is $0.72/r$. Note how closely the rule approximates the values in Table 2.1. With a 0.10 time value factor, an investment will double in value every 7.3 years. The rule of thumb gives 7.2 years.

Frequently, to make business decisions, instead of computing future values we will want to work with present values.

Time Indifference: Present Value

Today, close to 100 percent of large corporations use some form of discounted cash flow (DCF) techniques in their capital budgeting (investment decision making). To perform a DCF analysis, we must find the present value equivalents of future sums of money. For example, if the firm will receive \$100 one year from now as a result of a decision, we want to find the present value equivalent of the \$100.00. Assuming that the money is worth (can be borrowed or lent at) 0.10 per year, the \$100.00 is worth \$90.91 now. The indifference can be shown by noting that \$90.91 invested to earn 0.10 will earn \$9.09 interest in one year; thus, the investor starting with \$90.91 will have \$100.00 at the end of the year. If the investor can both borrow and lend funds at 0.10, the investor would be indifferent to receiving \$100.00 at the end of the year or \$90.91 at the beginning of the year.

If the 0.10 interest rate applies for two periods, the investor would be indifferent to receiving \$82.64 today or \$100 two years from today. If the \$82.64 is invested to earn 0.10 per year, the investor will have \$90.91 after one year and \$100 at the end of two years.

The unit of time can be different from a year, but the unit of time for which the interest rate is measured must be the same as the unit of time for measuring the timing of the cash flows. For example, the 0.10 used in the example is defined as the interest rate per year and is applied to a period of one year.

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Starting with equation (2.1), we have

$$FV = PV(1 + r)^n. \tag{2.1}$$

Dividing both sides of equation (2.1) by $(1 + r)^n$, we obtain

$$PV = \frac{FV}{(1 + r)^n}. \tag{2.2}$$

Using C_n to denote the cash flow at the end of period n and r to denote the time value of money, we find that the present value, PV, of C_n is

$$PV = \frac{C_n}{(1 + r)^n} \tag{2.3}$$

or, equivalently,

$$PV = C_n(1 + r)^{-n}, \tag{2.4}$$

where $(1+r)^{-n}$ is the present value of \$1 to be received at the end of period n when the time value of money is r. The term $(1 + r)^{-n}$ is called the present value factor, and its value for various combinations of time periods and interest rates is found in Table A of the Appendix to the book. Any hand calculator with the capability to compute y^x can be used to compute present value factors directly. If y^x is used, then $y = 1 + r$ and $x = n$. First, y^x is found, and then the reciprocal is taken to determine the present value factor. For example, to find the present value factor for $r = 0.10$, $n = 5$ using a typical calculator, we would place 1.10 in the calculator, press the y^x button, insert 5, press the "equals" button, and then the reciprocal to find 0.62092.

Example 2.1

What is the present value of \$1.00 to be received three time periods from now if the time value of money is 0.10 per period?

In Table A at the end of the book, the 0.10 column and the line opposite n equal to 3 gives 0.7513. If you invest \$0.7513 to earn 0.10 per year, after three years you will have \$1.00. Thus, $(1.10)^{-3} = 0.7513$.

What is the present value of \$100.00 to be received three time periods from now if the time value of money is 0.10? Since $(1.10)^{-3} = 0.7513$, the present

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value of \$100 is

$$PV = \$100(0.7513) = \$75.13.$$

If \$75.13 is invested at time 0, the following interest growth takes place with a 0.10 interest rate.

Time	**Investment at Beginning of Period**	**Interest**	**Investment at End of Period**
0	\$75.13	\$7.513	\$ 82.643
1	82.643	8.264	90.907
2	90.907	9.091	100.000

If investors can earn 0.10 per period and can borrow at 0.10, then they are indifferent to \$75.13 received at time 0 or \$100 at time 3.

The present value of a series of cash flows is the sum of the present values of each of the components.

Example 2.2

What is the present value of two cash flows, \$100.00 and \$200.00, to be received at the end of one and two periods from now, respectively, if the time value of money is 0.10?

Period t	**Cash Flow X_t**	**Present Value Factors PVF (t, 0.10)**	**Present Value PV**
1	\$100	0.9091	\$ 90.91
2	200	0.8264	165.28
		Total present value using 0.10 =	\$256.19

Present Value of an Annuity

Frequently, the evaluation of alternatives will involve a series of equal payments spaced equally through time. Such a series is called an annuity. The present value of an annuity of \$1 per period for n periods with the first payment one period from now is the sum of the present values of each dollar to be received:

$$\mathrm{B}(n, r) = \frac{1}{(1+r)} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \cdots + \frac{1}{(1+r)^n}, \qquad (2.5)$$

where $\mathrm{B}(n, r)$ is the symbolic representation for an annuity of n periods and an interest rate of r (first cash flow one period from now). It can be shown (see

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Appendix 2.1) at the end of this chapter that

$$B(n, r) = \frac{1 - (1 + r)^{-n}}{r}. \tag{2.6}$$

Equation (2.6) is for an annuity in arrears. Appendix Table B at the end of the book gives the present values of annuities of \$1 per period for different values of r and n. The present value of an annuity can also be computed directly using many hand calculators, or a personal computer.

If C dollars are received each period instead of \$1, we can multiply equation (2.6) by C to obtain the present value of C dollars per period. That is, for an annuity for C dollars per period, the present value is

$$PV = C \times B(n, r). \tag{2.7}$$

Example 2.3

The ABC Company is to receive \$1 a period for three periods, the first payment to be received one period from now. The time value factor is 0.10. Compute the present value of the annuity.

There are three equivalent solutions:

a. From Appendix Table B,

$$B(3, 0.10) = 2.4869.$$

b. Using equation (2.6) and Appendix Table A,

$$B(3, 0.10) = \frac{1 - (1 + r)^{-n}}{r} = \frac{1 - 0.7513}{0.10} = \frac{0.2487}{0.10} = 2.487.$$

c. Adding the first three entries in the 0.10 column in Appendix Table A,

$$\begin{aligned} (1.10)^{-1} &= 0.9091 \\ (1.10)^{-2} &= 0.8264 \\ (1.10)^{-3} &= \underline{0.7513} \\ B(3, 0.10) &= 2.4868. \end{aligned}$$

If, instead of \$1 per period, the amount is \$100.00, then using equation (2.7), we would multiply \$2.487 by \$100.00 and obtain \$248.70.

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An Annuity

When the first payment is at time 1, we have an annuity in arrears (also called an ordinary annuity). When the payment occurs at the beginning of each period, we have an annuity due (also called an annuity in advance). Equation (2.6) gives the present value of an annuity in arrears:

$$B(n, r) = \frac{1 - (1 + r)^{-n}}{r}. \tag{2.6}$$

If we have $(n + 1)$ payments of \$1 each period with the first payment taking place immediately, we would merely add \$1 to the value of equation (2.6). Thus, if B(3, 0.10) equals \$2.4868, a four-payment annuity with the first payment at time 0 would have a present value of \$3.4868. An n period annuity due is nothing more than an $(n - 1)$ period annuity in arrears plus the initial payment.

Present Value of a Perpetuity

A perpetuity is an annuity that goes on forever (an infinite sequence). If we let n of equation (2.6) go to infinity, so that the annuity becomes a perpetuity, then the $(1 + r)^{-n}$ term goes to zero, and the present value of the perpetuity using equation (2.6) becomes

$$B(\infty, r) = \frac{1}{r}. \tag{2.8}$$

Thus, if $r = 0.10$ and the series of cash receipts of \$1.00 per period is infinitely long, investors would pay \$10.00 for the infinite series. They would not pay \$11.00, since they could invest that \$11.00 elsewhere and earn \$1.10 per period at the going rate of interest, which is better than \$1.00 per period. Investors would like to obtain the investment for \$9.00, but no rational issuer of the security would commit to pay \$1.00 per period in return for \$9.00 when \$10.00 could be obtained from other lenders for the same commitment.

Although perpetuities are seldom a part of real-life problems, they are useful, since they allow us to determine the value of extreme cases. For example, if $r = 0.10$, we may not know the present value of \$1 per period for 50 time periods, but we do know that it is only a small amount less than \$10 since the present value of a perpetuity of \$1 per period is \$10 and 50 years is close enough to being a perpetuity for us to use \$10 as an approximate present value:

$$B(\infty, r) = \frac{1}{r} = \frac{1}{0.10} = \$10.$$

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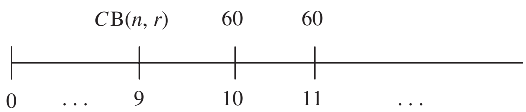
A Flexible Tool

We now have the tools to solve a wide range of time value problems that have not been described. While we could introduce other formulas, we prefer to adapt the three basic formulas that have been introduced.

For example, if a \$60-per-year annuity for 20 years were to have its first payment at time 10 and if the interest rate is 0.10, the present value is

$$\begin{aligned} PV &= C\mathrm{B}(n, r)(1+r)^{-t} = 60\mathrm{B}(20, 0.10)(1.10)^{-9} \\ &= 60\,(8.5136)(0.4241) = \$216.64. \end{aligned}$$



Note that if the first annuity payment is at time 10, we only have to discount the annuity for nine years to find the present value since B(10, r) gives the annuity present value as of time 9 and the first payment at time 10.

We want to determine how much we would have at time 29 if we saved \$60 per year for 20 years, with the first amount saved starting at time 10. Above we obtained \$216.64 for the present value of \$60 per year. The future value (time 29) of \$216.64 is

$$\text{Future value} = \$216.64(1.10)^{29} = \$216.64(15.8631) = \$3{,}437.$$

Now assume the \$60 annuity starting at time 10 is a perpetuity. The present value at time 9 is $\$60/0.10 = \$600$. The present value of the \$600 at time 0 is

$$PV = \$600(1.10)^{-9} = \$600(0.4241) = \$254.46.$$

Another approach to solving for the present value of the annuity is to compute the present value of a perpetuity and subtract the present value of a nine-period annuity:

$$PV = \$600 - \$60\mathrm{B}(9, 0.10) = \$600 - \$60(5.7590) = \$254.46.$$

The basic time-discounting tools are very flexible.

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Annual Equivalent Amounts

In many situations we want to determine the annual equivalent of a given sum. For example, what is the annual equivalent over 20 years of $100,000 received today if the time value of money is 10 percent?

Solve equation (2.7) for the annual cash flow:

$$C = \frac{PV}{\mathrm{B}(n, r)}. \tag{2.9}$$

That is, to find the annual equivalent, C, of a present sum, PV, that sum is divided by the annuity factor, $\mathrm{B}(n, r)$, where r is the time value of money and n is the number of years over which the annual equivalent is to be determined.

Calculations of this type are particularly useful in the management of financial institutions such as insurance companies or banks, where customers make periodic payments over an extended time period in return for a lump-sum immediate loan.

Example 2.4

The ABC Company wishes to borrow $10,000 from the City Bank, repayable in three annual installments (the first one due one year from now). If the bank charges 0.10 interest, what will be the annual payments?

From equation (2.9),

$$\begin{aligned} C &= \frac{\$10{,}000}{\mathrm{B}(3, 0.10)} \\ &= \frac{\$10{,}000}{2.4869} = \$4{,}021 \end{aligned}$$

and the loan amortization schedule is

(1) Time	(2) Beginning Balance	(3) Interest 0.10 of (2)	(4) Payment	(5) = (2) + (3) − (4) Ending Balance
0	$10,000	$1,000	$4,021	$6,979
1	6,979	698	4,021	3,656
2	3,656	366	4,021	0

The loan amortization schedule starts with the initial amount owed. Column 3 shows the interest on the amount owed at the beginning of the period (column 2). Column 4 is the payment to pay the debt and column 5 shows the ending debt

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balance. The ending debt balance is equal to the beginning debt balance plus the period's interest less the debt payment.

The process is repeated for the life of the debt. If the present value of the debt payments is equal to the initial beginning balance (as it will be using the effective cost of debt to compute the present value), the ending balance after the last debt payment will be equal to zero.

Conclusions

Most investment analyses performed by a company are made on the basis of annual cash flows. Finer divisions of time are usually unwarranted in light of the approximate nature of the cash flow estimates. Some firms use present value tables that assume the cash flows are distributed evenly over the year or occur at the midpoint of the year in question rather than at the end of the year, as do the present value tables at the end of this book. Such refinements add little to the substance of discounted cash flow analysis and are not likely to alter materially any investment decision obtained from using the "end-of-year" assumption.

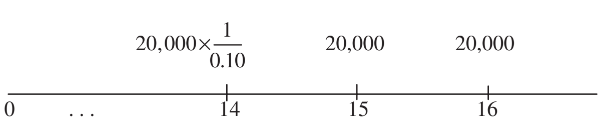
Most financial decision making can be reduced to evaluating incremental or alternative cash flows. There are three steps in the analysis. First, the relevant incremental cash flows must be estimated. Second, there must be some means of dealing with uncertainty if the cash flows are uncertain. Third, there must be some way to take into consideration the time value of money. The material in this chapter is essential for dealing with the time value of money to determine the present and future values of sums of money to be received or paid at various times.

Review Problems

Review Problem 2.1

Exactly 15 years from now, Jones will start receiving a pension of $20,000 a year. The payments will continue forever. How much is the pension worth now, assuming that the appropriate discount rate is 0.10 per year?

Solution to Review Problem 2.1



The present value at time 14 is $200,000.

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The present value at time 0 is

$$\begin{aligned}\$200{,}000(1.10)^{-14} &= \$200{,}000(0.26333)\\ &= \$52{,}666.\end{aligned}$$

Review Problem 2.2

(a) Twelve rental payments of \$1,000 will be paid monthly at the end of each month. The monthly interest rate is 0.01. What is the present value of the payments?
(b) If the payments are at the beginning of each month, what is the present value?

Solution to Review Problem 2.2

(a) $\$1{,}000B(12, 0.01) = \$1{,}000(11.2551) = \$11{,}255.$
(b) $\$1{,}000[B(11, 0.01) + 1] = \$1{,}000(10.3676 + 1) = \$11{,}368$
or $\$11{,}255(1.01) = \$11{,}368.$

Questions and Problems

1. Assume a 0.05-per-year time value of money. Compute the value of \$100 (a) received 1 year from now; (b) received immediately; (c) received at the end of 5 years; (d) received at the beginning of the sixth year; (e) received at the end of 50 years; (f) received at the end of 50 years, but with an interest rate of 0.10.
2. Assume that the interest rate is 0.10. Compute the present value of \$1 per year for four years (first payment one year from now) using three different methods (Appendix Table A, Appendix Table B, and an equation).
3. Assume a 0.05 time value of money. Compute the value of the following series of payments of \$100 a year received for (a) five years, the first payment received one year from now; (b) four years, the first of five payments received immediately; (c) ten years, the first payment received one year from now; (d) nine years, the first of ten payments received immediately.
4. Assume a 0.05 time value of money. The sum of \$100 received immediately is equivalent to what quantity received in 10 equal annual payments, the first to be received one year from now? What would be the annual amount if the first of 10 payments were received immediately?
5. Assume a 0.05 time value of money. We have a debt to pay and are given a choice of paying \$1,000 now or some amount X five years from now. What is the maximum amount that X can be for us to be willing to defer payment for five years?

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6. We can make an immediate payment now of $10,000 or pay equal amounts of *R* for the next five years (first payment due one year from now). With a time value of money of 0.05, what is the maximum value of *R* that we would be willing to accept?
7. If the interest rate per month is 0.05, compounded quarterly, what is the annual equivalent rate?
8. If a firm borrowed $100,000 for one year and paid back $9,455.96 per month, what is the cost of the debt?
9. A firm can save $10,000 per year for 15 years. If the time value of money is 0.10, how much better off will the firm be after the 15 years if it accomplishes the saving?
10. If the time value of money is 0.10, how much do you have to save per year for 20 years to have $50,000 per year for perpetuity? Assume that the first deposit is immediate and that the first payment will be at the beginning of the twenty-first year.
11. If $100 earns 0.08 per year, how much will the investor have at the end of 10 years? What is the present value of $100 due in 10 years if money is worth 0.08?
12. What is the present value of $20 per year for perpetuity if money is worth 0.10?
13. Refer to Problem 12. If the first payment is to be received in 11 years, what is the series of payments worth today?
14. You are the loan officer of a bank. The ABC Company wants to borrow $100,000 and repay it with four equal annual payments (first payment due one year from now). You decide that the ABC Company should pay 0.10 per year on the loan.

    a. What is the annual payment?
    b. Complete the following debt amortization table:

Period	Amount Owed (beginning of year)	Interest	Principal	Amount Owed (end of year)
1	$100,000			
2				
3				
4				

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c. What would be the annual payment if the first of four equal payments is due immediately?

15. a. If the interest rate per month is 0.02, compounded monthly, what is the annual effective equivalent rate?
    b. How much do you have to save per year for 20 years in order to have \$50,000 per year for perpetuity? $r = 0.10$. The first \$50,000 payment will be received at time 21.
    c. If \$100 will grow into \$120 in one year, what is the continuous rate of growth?
16. Assume a 0.10 interest rate. How much is a perpetuity of \$1,000 per year worth?
17. Assume a 0.10 interest rate (you can borrow and lend at that rate). Specify which you would prefer:
    a. \$10,000 in cash or \$1,000 per year for perpetuity (first payment received at the end of the first period).
    b. \$10,000 in cash or \$1,100 per year for perpetuity (first payment received at the end of the first period).
    c. \$10,000 in cash or \$900 per year for perpetuity (first payment received at the beginning of the first period).
18. a. What would be the annual payments on an 8% per annum installment loan of \$1,000 from a credit union with repayment over three years?
    b. Write out the amortization schedule for the loan.
    c. Now suppose that the payments are to be made on a semiannual basis; what would the semiannual payments be? Assume the 0.08 is a nominal rate.
    d. Is the total paid in case (c) less or more than in the former case? Why?
19. a. How much do you have to save per year (at the end of each year) for 40 years in order to have \$10,000 per year for perpetuity, first receipt starting in year 41? Use 0.10 as the time value factor.
    b. If the interest rate being earned is 0.04 per quarter, compounded quarterly, what is the annual equivalent rate?
20. a. We can make an immediate payment now of \$10,000 or pay equal amounts of R for the next four years (first payment due one year from now). With a time value of money of 0.10, what is the maximum value of payment that we would be willing to make?
    b. Now assume that the first of the *five* payments is *immediate*. What is the maximum value of payment we would be willing to make?
21. The XYZ Company has borrowed \$100,000. Payments will be made over a four-year period (first payment at the end of the first year). The bank charges interest of 0.20 per year.

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a. The annual payment will be _____.
b. The debt amortization schedule is:

Amount Owed (Beginning of Period)	Interest	Principal
1. $100,000		
2.		
3.		
4.		

c. If there are five payments with the first payment made at the moment of borrowing, the annual payment will be _______.

22. The XYZ Company has borrowed $40,000. Equal payments will be made over a three-year period (first payment at the end of the first year). The bank charges interest of 0.15 per year.

a. The annual payment will be _____.
b. The debt amortization schedule is:

Amount Owed (Beginning of Period)	Interest	Principal
1. $40,000		
2.		
3.		
4.		

c. If there are four payments with the first payment made at the moment of borrowing, the annual payment will be _______.

Appendix 2.1: The Derivation of an Annuity Formula

Let

$$
\begin{aligned}
r &= \text{the time value of money per period} \\
n &= \text{the number of time periods} \\
\mathrm{B}(n, r) &= \text{the present value of an annuity for } n \text{ periods, } r \text{ interest rate,} \\
&\quad\ \text{with the first payment one year from time zero.}
\end{aligned}
$$

In the following table, each entry in column (1) gives the present value of $1 received at the end of the period indicated in the column headed "Time". The sum of the items in this column is $\mathrm{B}(n, r)$. Each entry in column (2) of this table gives the item in that row of column 1 multiplied by $(1 + r)$. The sum of the items in this column is $(1 + r)\mathrm{B}(n, r)$. Note that $(1 + r)^0 = 1$ and that all except two of the

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amounts are in both columns. Taking the difference between the sum of the two columns and solving for $B(n, r)$ gives the formula we wish to derive.

Time	**(1)**	**(2)**
1	$(1+r)^{-1}$	$(1+r)^{0}$
2	$(1+r)^{-2}$	$(1+r)^{-1}$
3	$(1+r)^{-3}$	$(1+r)^{-2}$
.	.	.
.	.	.
.	.	.
$n-1$	$(1+r)^{-n+1}$	$(1+r)^{-n+2}$
n	$(1+r)^{-n}$	$(1+r)^{-n+1}$
	$B(n, r)$	$(1+r)B(n, r)$

Column (2) minus column (1) yields

$$(1+r)B(n, r) - B(n, r) = 1 - (1+r)^{-n}.$$

Simplifying the left-hand side,

$$rB(n, r) = 1 - (1+r)^{-n}$$

and, dividing by r,

$$B(n, r) = \frac{1 - (1+r)^{-n}}{r}.$$

Appendix Table B of this book gives the values of $B(n, r)$, the present value of an annuity of \$1 per period. If the annuity is for \$$R$, we multiply the value obtained from the table by R.

If n is very large (let n approach infinity), we have the present value for a perpetuity of \$1 per period:

$$B(\infty, r) = \frac{1}{r}.$$

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